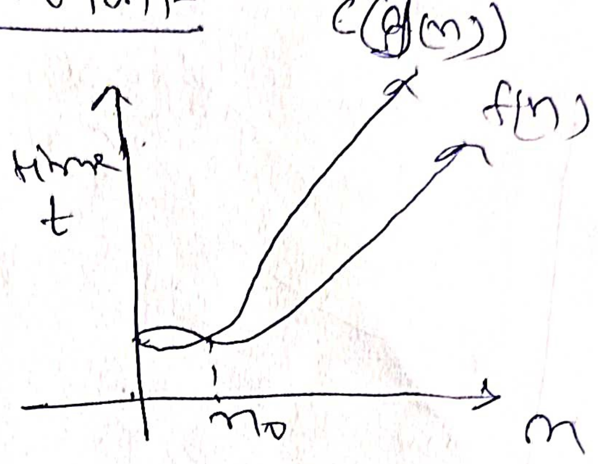
1

Asymptotic notation -

① Big(Oh) - 'O'



$f(n) \le c\, g(n)$

$n \ge n_0$

$c > 0 \quad n_0 \ge 1$

$\boxed{f(n) = O(g(n))}$

where c, n are real no.

Ex $\boxed{f(n) = 3n+2}$, $\boxed{g(n) = n}$

g(n) will be n^2, n^3, n^n & 2^n

$f(n) = O(g(n))$

$f(n) \le c\, g(n)$, where $c > 0$, $n_0 \ge 1$

but n is suitable and useful

$3n+2 \le cn$

Let $c = 4$ $3n+2 \le 4n$

$\boxed{n \ge 2}$

so $\boxed{f(n) = O(n)}$

Remark

avg bound

$1 < \log n < \sqrt{n} < \boxed{n} < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n < n^n$

Lower Bound Upper Bound

$f(n) = 3n+2$

closest fun

$3n+2 \le 10n$ True $f(n) = O(n)$ True

$\le 7n$ True $f(n) = O(n^2)$ True

$\le 100n$ " $f(n) = O(n^3)$ "

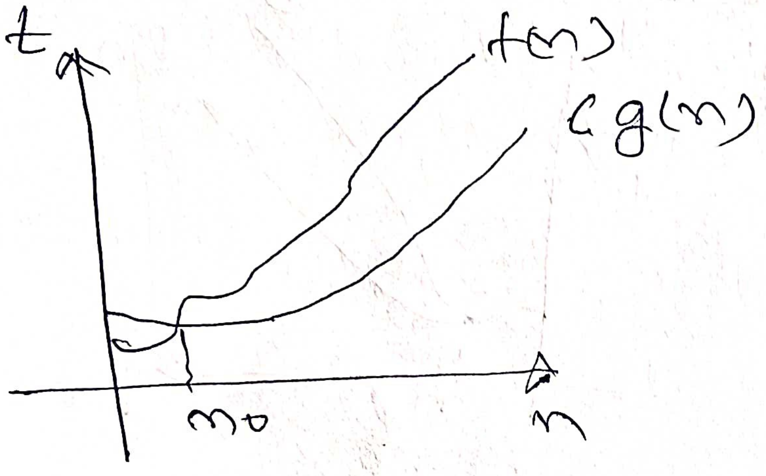
$3n+2 \le 3n+2n$ $f(n) = O(2^n)$ "

$3n+2 \le 5n$ True $f(n) = O(\log n)$ False

$\le 2n^2+3n^2$ "

$\le 5n^2$ "

Big Omega 'Ω'



$$\boxed{f(n) = \Omega\, g(n)}$$

if $f(n) \geq c\, g(n)$

where $c > 0$, $n_0 \geq 1$

$n > n_0$

Ex $f(n) = 3n+2$ $g(n) = n$

$\because$ $f(n) \geq c\, g(n)$

$3n+2 \geq 1 \cdot n$ $\boxed{c=1}$

Here $3n+2 = \Omega(n)$

Note –

Let $f(n) = 3n+2$ $g(n) = n^2$

check that weather it satisfy Ω condition

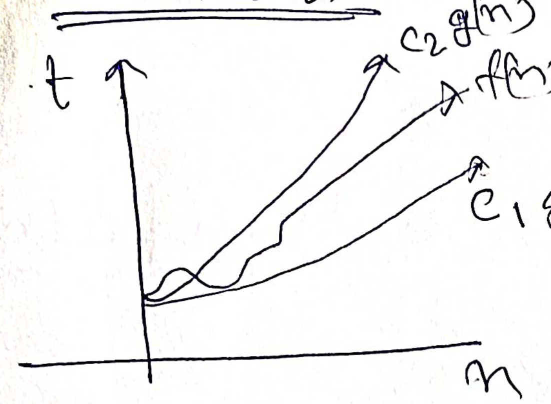
Soln $3n+2 \geq cn^2$

if $c=1$ it not satisfy

it satisfy only values less than n like

$\log n$, $\log \log n$

Theta Θ



$\boxed{f(n) = \Theta(g(n))}$

if $c_1 g(n) \le f(n) \le c_2 g(n)$
$c_1, c_2 > 0$, $n > n_0$
and $\boxed{n_0 \ge 1}$

Ex $f(n) = 3n+2$ $g(n) = n$

(a) $f(n) \le c_2 g(n)$

$3n+2 \le 4n$ $\therefore c_2 = 4$

(b) $f(n) \ge c_1 g(n)$

$3n+2 \ge 1n$

~~i~~ both condition a & b
are satisfy
so $\boxed{f(n) = \Theta(n)}$

Ex

$1 < \log n < \sqrt{n} < n < n\log n < n^2 < \ldots < 2^n < 3^n \ldots < 2^n$

(a) $f(n) = 2n^2+3n+4$

$2n^2+3n+4 \le 2n^2+3n^2+4n^2$
$\le 9n^2$ for $n \ge 1$

$f(n)$ c $g(n)$

$f(n) = O(n^2)$

(b) if $2n^2+3n+4 \geq 1 \times n^2$

$f(n)$ ... c ... $g(n^2)$

$f(n) \geq c\, g(n)$ so this is $\boxed{\Omega(n^2)}$

(c)

$1 \times n^2 \leq 2n^2+3n+4 \leq 9n^2$

this is $\boxed{\Theta(n^2)}$

Ex $f(n) = n^2 \log n + n$

$1 \times n^2 \log n \leq n^2 \log n \leq 10\, n^2 \log n$

so it is $O(n^2 \log n)$ $\Omega(n^2 \log n)$

and $\Theta(n^2 \log n)$

we can classify it in between n^2 and n^3

Ex $f(n) = n!$

$\Rightarrow n(n-1)(n-2) \ldots 3 \times 2 \times 1$

$1 \times 1 \times 1 \ldots \times 1 \leq 1 \times 2 \times 3 \times \ldots \times n \leq n \times n \times n \times n \ldots \times n$

$1 \leq n! \leq n^n$

it is only $\boxed{\begin{array}{c} O(n^n) \\ \Omega(1) \end{array}}$

$\because$ both side of $n!$ are not same so we cannot find Θ

Ex

$$f(n) = \log n!$$

$$\log(1 \times 1 \times \cdots \times 1) \le \log(1 \times 2 \times 3 \times \cdots \times n) \le \log(n \times n \times \cdots \times n)$$

$$1 \le \log n! \le \log n^n$$

$$\Downarrow$$

$$\boxed{1 \le \log n! \le n \log n}$$

$$\boxed{\begin{array}{c} O(n \log n) \\ \Omega(1) \end{array}}$$ Ans